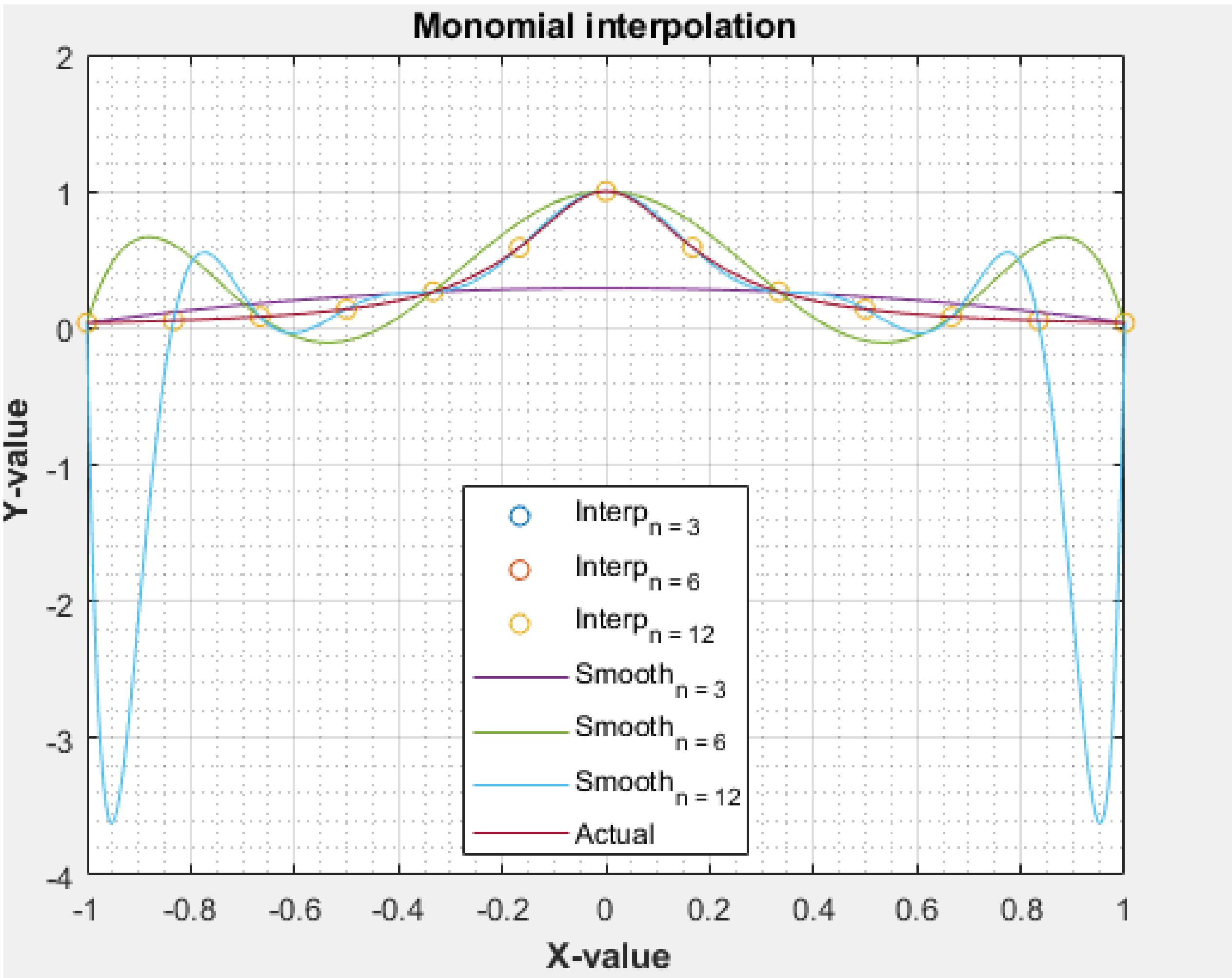
HW 8

MATH 375

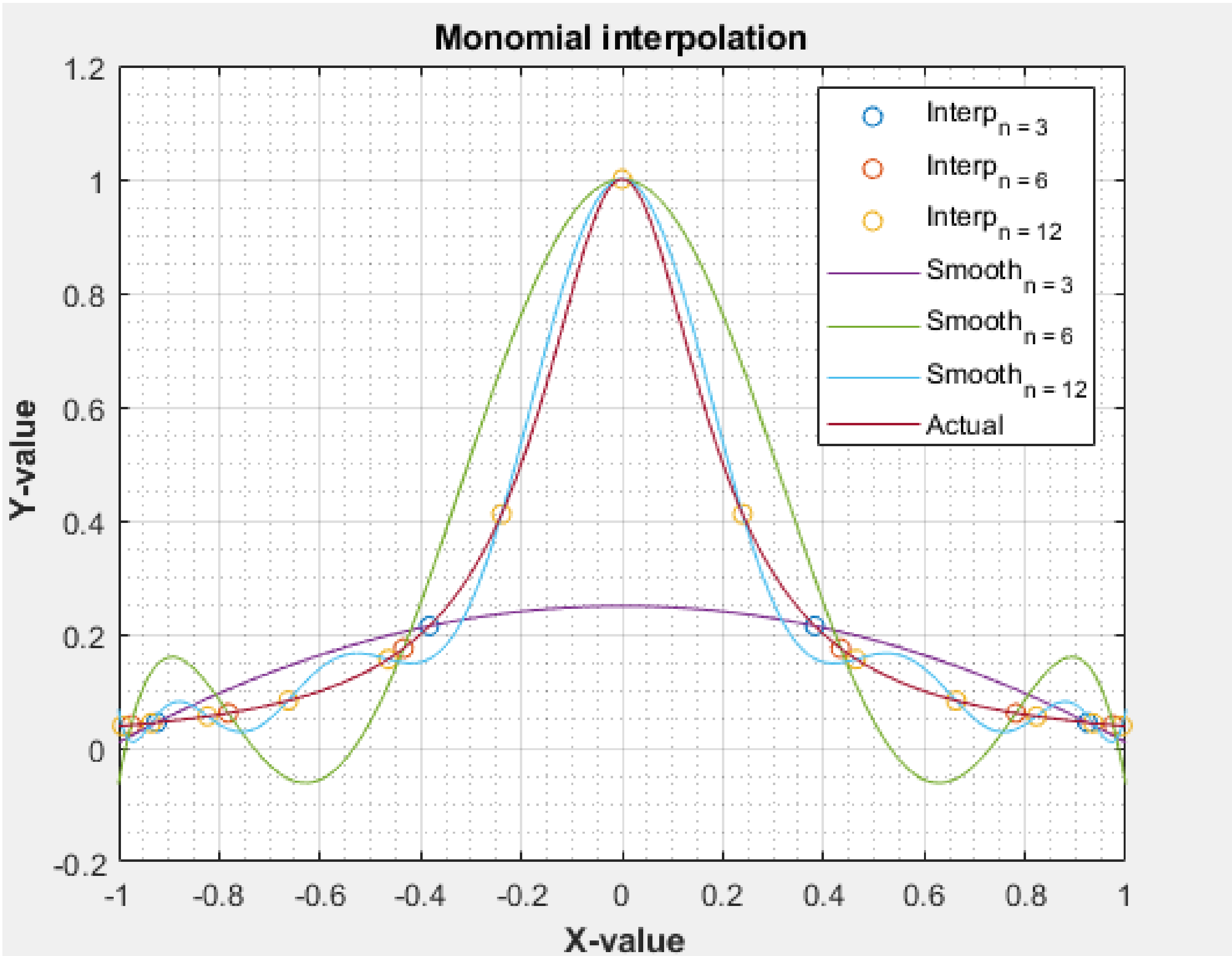
Michael Tanguay

TYPED SECTION

2.)

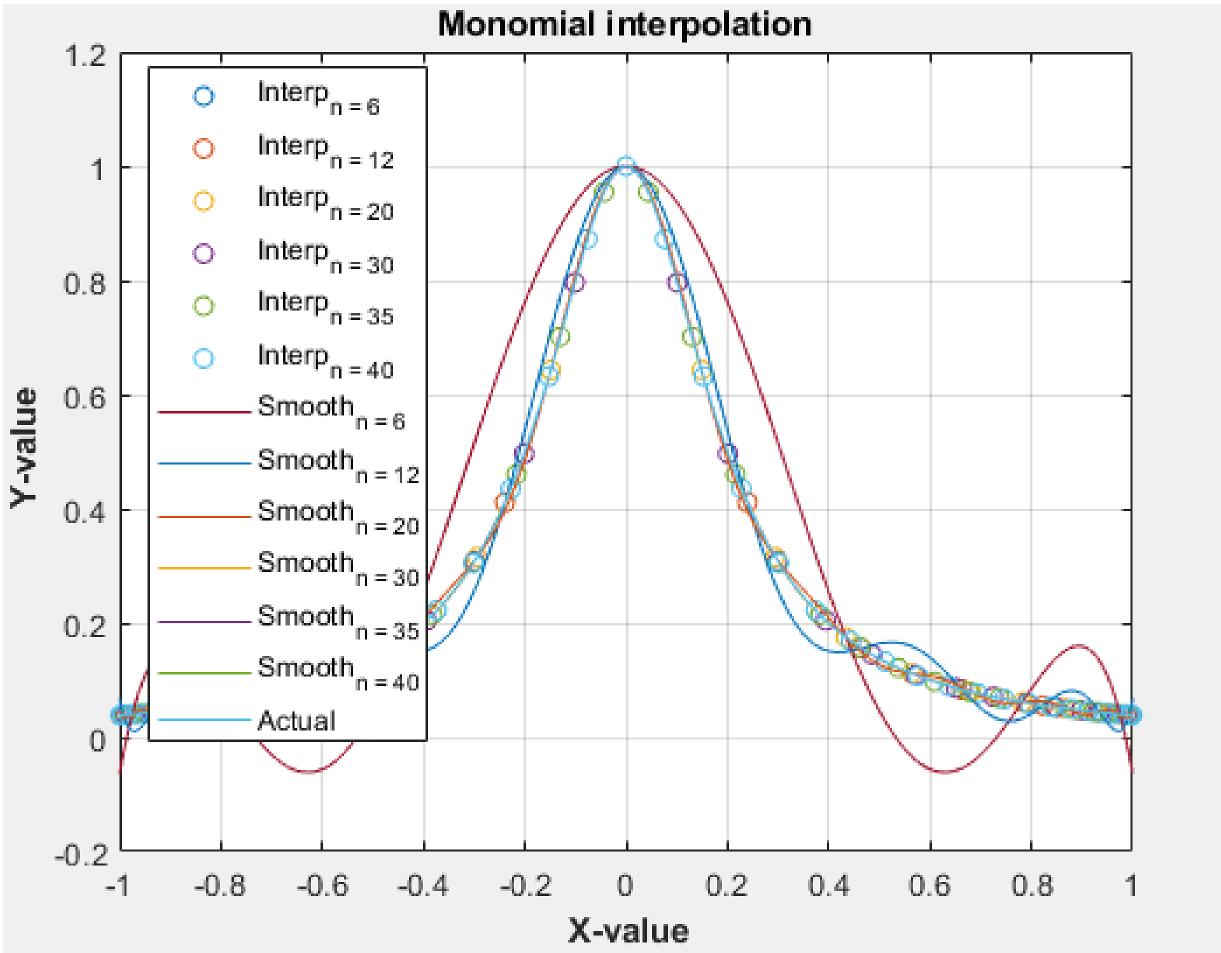


3.)



Monomial interpolation
Y-value
X-value
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Interp$_{n=3}$
Interp$_{n=6}$
Interp$_{n=12}$
Smooth$_{n=3}$
Smooth$_{n=6}$
Smooth$_{n=12}$
Actual

3.ii)



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1
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0
-0.2
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-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
$\text{Interp}_{n=6}$
$\text{Interp}_{n=12}$
$\text{Interp}_{n=20}$
$\text{Interp}_{n=30}$
$\text{Interp}_{n=35}$
$\text{Interp}_{n=40}$
$\text{Smooth}_{n=6}$
$\text{Smooth}_{n=12}$
$\text{Smooth}_{n=20}$
$\text{Smooth}_{n=30}$
$\text{Smooth}_{n=35}$
$\text{Smooth}_{n=40}$
Actual

Problem 1

```
clear, clc, close all
syms x y z

y = simplify((x-3)*(x-7) - 3*(x-2)*(x-7) + (x-2)*(x-3))
y2 = simplifyFraction(5 + 7*(x-2) + -(x-2)*(x-3))

fplot(y); hold on; fplot(y2)
```

Problem 2

```
%main program for P2
clear, clc, close all, format short
n = [3,6,12];
for z = 1:length(n)
   h = 2/(n(z));
   x = [-1:h:1];
   y = 1./(1 + 25.*x.^2);
   a{z} = (P2(x,y));
   for i = 1:length(x)
      if norm((abs(polyval(a{z},x(i)) - y(i))./y(i))) > 0.001
         fprintf('For n = %d, the relative error has exceeded 0.001\n', n(z))
         break;
      end
   end
   figure(1); plot(x,polyval(a{z},x), 'o'); grid on; grid minor; hold on;
end
x = [-1:.001:1];
for z = 1:length(n)
  plot(x,polyval(a{z},x))
end
for z = 1:length(n)*2 + 1
   if z <= length(n)
      Legend{z} = sprintf('Interp_{n = %d}',n(z));
   elseif z == length(n)*2 + 1
      Legend{z} = sprintf('Actual');
   else
      Legend{z} = sprintf('Smooth_{n = %d}',n(z-length(n)));
   end
end
y = 1./(1 + 25.*x.^2);   plot(x,y);
legend(Legend,'Location', 'best'); title('Monomial interpolation'); xlabel('\bf X-value'); ylabel('\bf Y-
…value');
```

Problem 3

```
%main program for P2
```

```
clear, clc, close all, format short
n = [3,6,12];
for z = 1:length(n)
   for i = 0:n(z)
        x(i+1) = cos((2*i+1)*pi/(2*n(z)+2));
   end
   length(x)
   y = 1./(1 + 25.*x.^2);
   a{z} = (P2(x,y));
   for i = 1:length(x)
      if norm((abs(polyval(a{z},x(i)) - y(i))./y(i))) > 0.001
         fprintf('For n = %d, the relative error has exceeded 0.001\n', n(z))
         break;
      end
   end
   figure(1); plot(x,polyval(a{z},x), 'o'); grid on; grid minor; hold on;
end
x = [-1:.001:1];
for z = 1:length(n)
  plot(x,polyval(a{z},x))
end
for z = 1:length(n)*2 + 1
   if z <= length(n)
      Legend{z} = sprintf('Interp_{n = %d}',n(z));
   elseif z == length(n)*2 + 1
      Legend{z} = sprintf('Actual');
   else
      Legend{z} = sprintf('Smooth_{n = %d}',n(z-length(n)));
   end
end
y = 1./(1 + 25.*x.^2);   plot(x,y);
legend(Legend,'Location', 'best'); title('Monomial interpolation'); xlabel('\bf X-value'); ylabel('\bf Y-
…value');
```

<u>Functions used for 2 and 3:</u>

```
%This answers 2 and 3
function [a] = interp_monomials(x,y)
for i = 1:length(x)
   for k = 1:length(x)
   M(k,i) = x(k)^(i-1);
   end
end
a = flip(linsolve(M,y')); end
```

WRITTEN SECTION

Michael Tamgong
Math 375 - HW 8
04/14/20

1a) $P(x) = \sum_{k=0}^{n} \ell_k(x) y_k = 5\ell_0 + 12\ell_1 + 20\ell_2$, where $\ell_k(x) = \prod_{i=0, i\neq k}^{n} \frac{x - x_i}{x_k - x_i}$

$\ell_0: \left(\frac{x-3}{2-3}\right)\left(\frac{x-7}{2-7}\right)$, $\ell_1: \left(\frac{x-2}{3-2}\right)\left(\frac{x-7}{3-7}\right)$, $\ell_2: \left(\frac{x-2}{7-2}\right)\left(\frac{x-3}{7-3}\right)$

$= \frac{(x-3)(x-7)}{(-1)(-5)}$, $= \frac{(x-2)(x-7)}{(1)(-4)}$, $= \frac{(x-2)(x-3)}{(5)(4)}$

$= \frac{(x-3)(x-7)}{5}$, $= \frac{(x-2)(x-7)}{-4}$, $= \frac{(x-2)(x-3)}{20}$

$P(x) = 5\frac{(x-3)(x-7)}{5} + \frac{12(x-2)(x-7)}{-4} + 20\frac{(x-2)(x-3)}{20}$

$= \boxed{(x-3)(x-7) - 3(x-2)(x-7) + (x-2)(x-3)} = -x^2 + 12x - 15$

1b) $P_2 = a_0 + a_1(x - x_0) + a_2(x-x_0)(x-x_1)$

$= a_0 + a_1(x-2) + a_2(x-2)(x-3)$

$a_0 = 5$, $a_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{12-5}{3-2} = 7$, $a_2 = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$

$a_2 = \frac{\frac{20-12}{7-3} - \frac{12-5}{3-2}}{7-2} = \frac{\frac{8}{4} - \frac{7}{1}}{7-2} = \frac{\frac{8}{4} - \frac{28}{4}}{5} = -1$

for the table:

x	$f[\cdot]$	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$
2	5		
		$\frac{12-5}{3-2} = 7$	
3	12		$\frac{\frac{8}{4} - 7}{7-2} = \frac{\frac{8}{4} - \frac{28}{4}}{5} = \frac{-5}{5} = -1$
		$\frac{20-12}{7-3} = \frac{8}{4}$	
7	20		

$P_2 = \boxed{5 + 7(x-2) - (x-2)(x-3)} = -x^2 + 12x - 15$

1c) $\boxed{-x^2 + 12x - 15 = -x^2 + 12x - 15.}$ See Matlab code for simplification.

2a.) $Va = y$

$$\Rightarrow \begin{bmatrix} 1 & x_0 & x_0^2 & \cdots & x_0^n \\ 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

2b.) See Matlab

2c.) See Matlab

2d.) See Written.

2e.) at $n = 33$, Rcond $= 1.278572 \times 10^{-16}$, Cond $= 7.82123 \times 10^{15}$ essentially means it is ill conditioned $\Rightarrow$ that is as $n \to \infty$ the Matrix becomes exponentially ill-conditioned and is primarily do to the fact the y is very sensitive to change as $n \to \infty$ and therefore is extremely difficult to find a solution vector a and thus making it impossible to find interpolation points.

3.) It has exceeded $n = 33$ and now fails at $n = 41$ at Rcond $= 1.471355 \times 10^{-16}$, Rcond $= 6.7465 \times 10^{15}$
SEE Plot in Written section